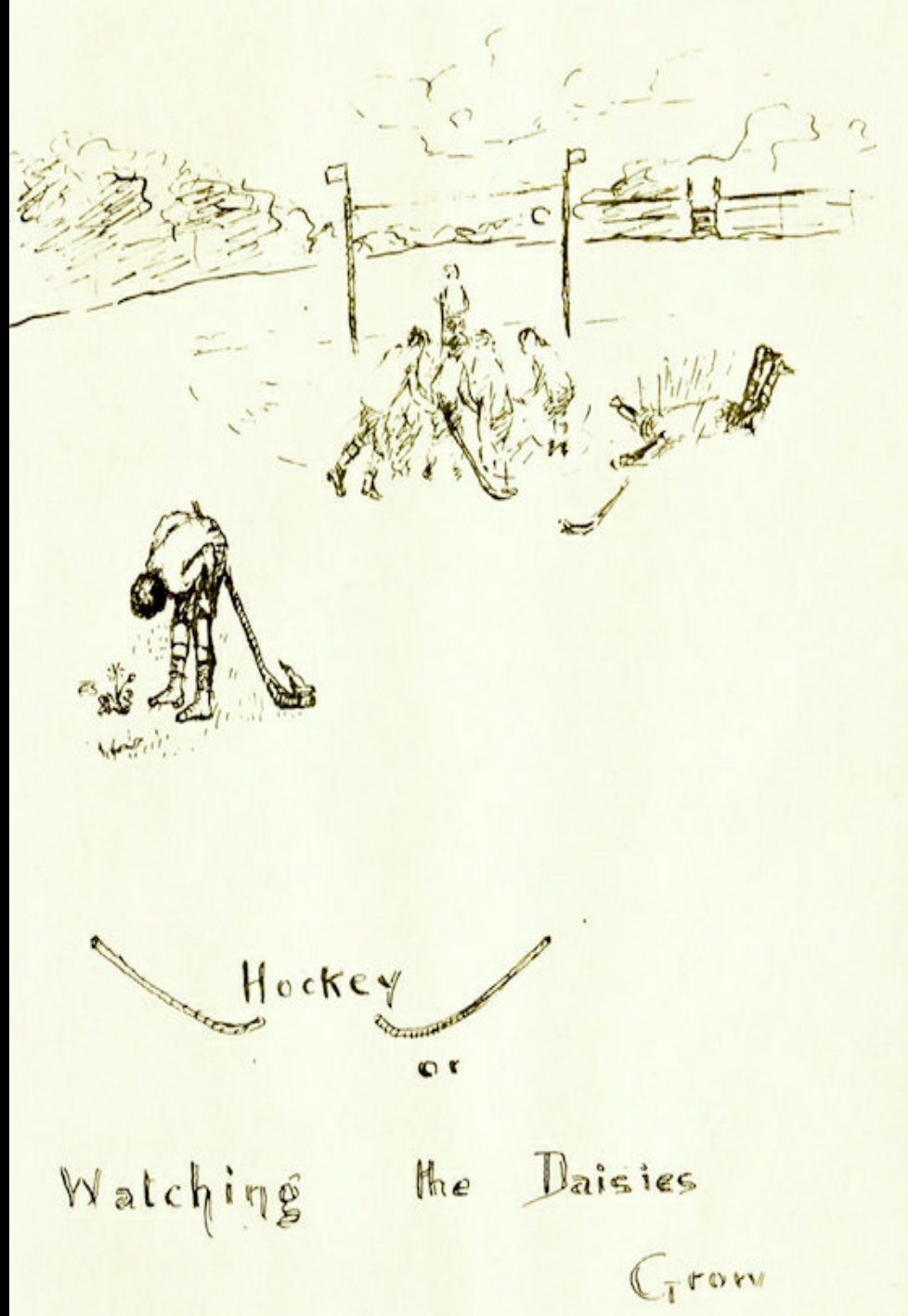




# Morphogenesis

Source: Google images



Source:  
*The mathematics of  
nature at the Alan  
Turing centenary*  
S. Barry Cooper and  
Philip K. Maini



# Alan Turing

University of Cambridge

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<input type="checkbox"/> <a href="#">The chemical basis of morphogenesis</a>	AM Turing Philosophical Transactions of the Royal Society of London. Series B ...	6003	1952
<input type="checkbox"/> <a href="#">Computing machinery and intelligence</a>	AM Turing Mind 59 (236), 433-460	5443	1950
<input type="checkbox"/> <a href="#">On computable numbers, with an application to the Entscheidungsproblem (1936)</a>	A Turing B. Jack Copeland, 58	5440	2004





# Alan Turing

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Title	<a href="#">The chemical basis of morphogenesis</a>	<a href="#">[PDF] from usp.br</a>
Authors	Alan Mathison Turing	
Publication date	1952/8/14	
Journal name	Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences	
Volume	237	
Issue	641	
Pages	37-72	
Publisher	The Royal Society	
Description	<p>Abstract It is suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis. Such a system, although it may originally be quite homogeneous, may later develop a pattern or structure due to an instability of the homogeneous equilibrium, which is triggered off by random disturbances. Such reaction-diffusion systems are considered in some detail in the case of an isolated ring of cells, a mathematically convenient, though ...</p>	

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AM Turing - Philosophical Transactions of the Royal Society of ..., 1952  
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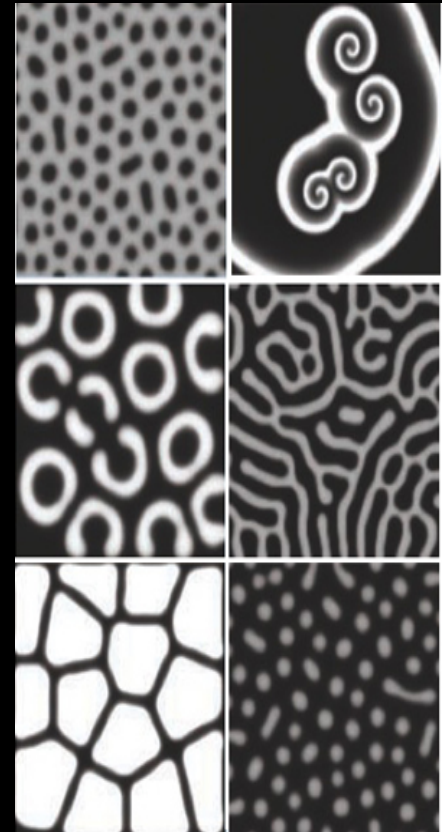
A. M. Turing

*Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, Vol. 237, No. 641. (Aug. 14, 1952), pp. 37-72.

- What might cause phenomenon of developmental patterns such as *phyllotaxis*.
- Turing proposed a theory of developmental pattern formation.

Reacting chemicals

diffusion



# Morphogens as chemical pre-patterns

It is suggested that a system of chemical substances, called morphogens, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis.

Turing's main hypothesis was that the resulting **gradient of chemical concentrations** could **cue differential cell growth** and result in **pattern formation**.



# Diffusion-driven instability

- A system of chemicals which is **stable** in the absence of diffusion becomes **unstable** in the presence of diffusion.

# A Reaction-diffusion Model

$$\frac{\partial u}{\partial t} = \gamma f(u, v) + D_u \nabla^2 u,$$

$$\frac{\partial v}{\partial t} = \gamma g(u, v) + D_v \nabla^2 v,$$

$$\frac{\partial f}{\partial u} + \frac{\partial g}{\partial v} < 0;$$

$$\frac{\partial f}{\partial u} \frac{\partial g}{\partial v} - \frac{\partial f}{\partial v} \frac{\partial g}{\partial u} > 0;$$

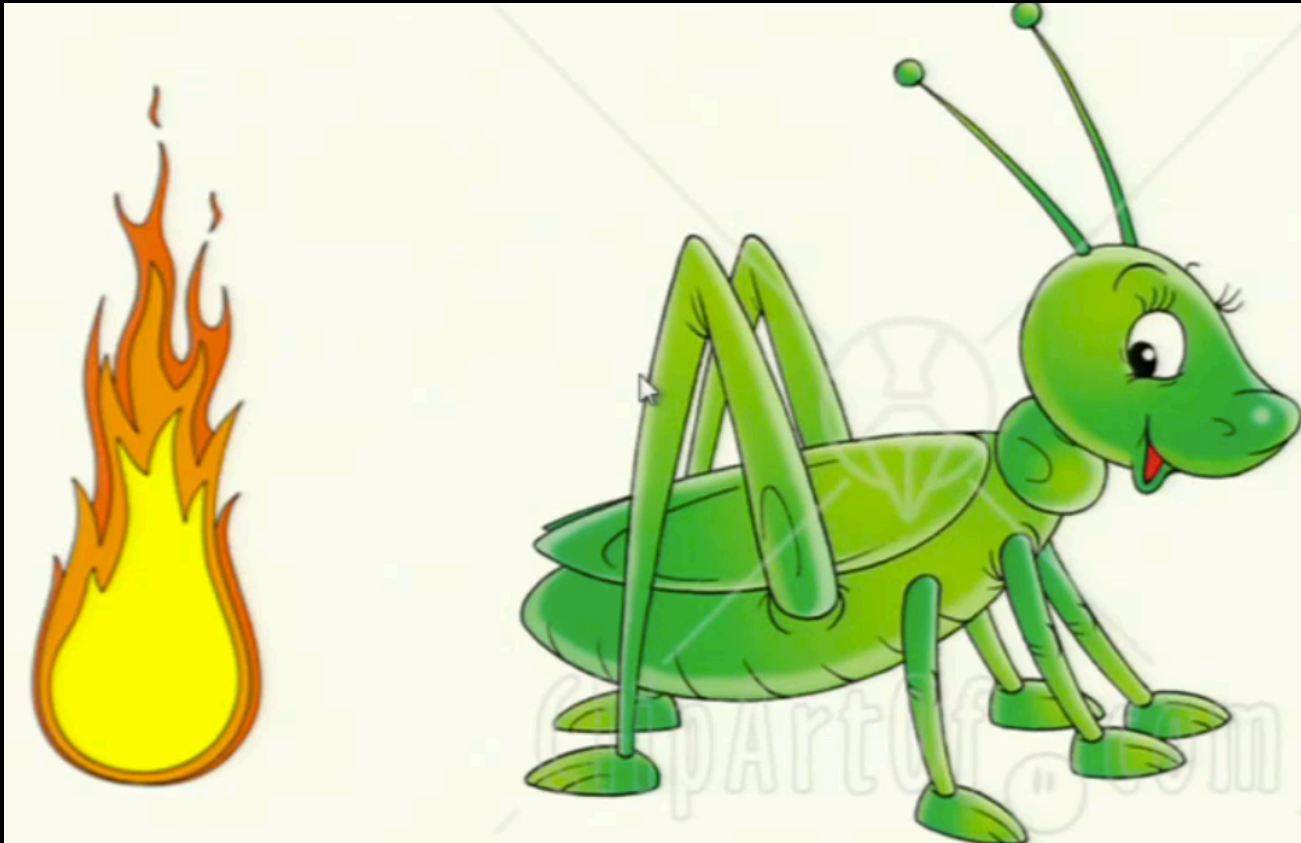
$$D_u \frac{\partial g}{\partial v} + D_v \frac{\partial f}{\partial u} > 0;$$

$$D_u \frac{\partial g}{\partial v} + D_v \frac{\partial f}{\partial u} > \sqrt{D_u D_v} \left( \frac{\partial f}{\partial u} \frac{\partial g}{\partial v} - \frac{\partial f}{\partial v} \frac{\partial g}{\partial u} \right)$$

A stability analysis of the steady states of the kinetics shows that to generate spatial patterns in  $u$  and  $v$ , it is necessary, among other things, that the inhibitor have a higher diffusion rate than the activator, that is  $D_v > D_u$ ;



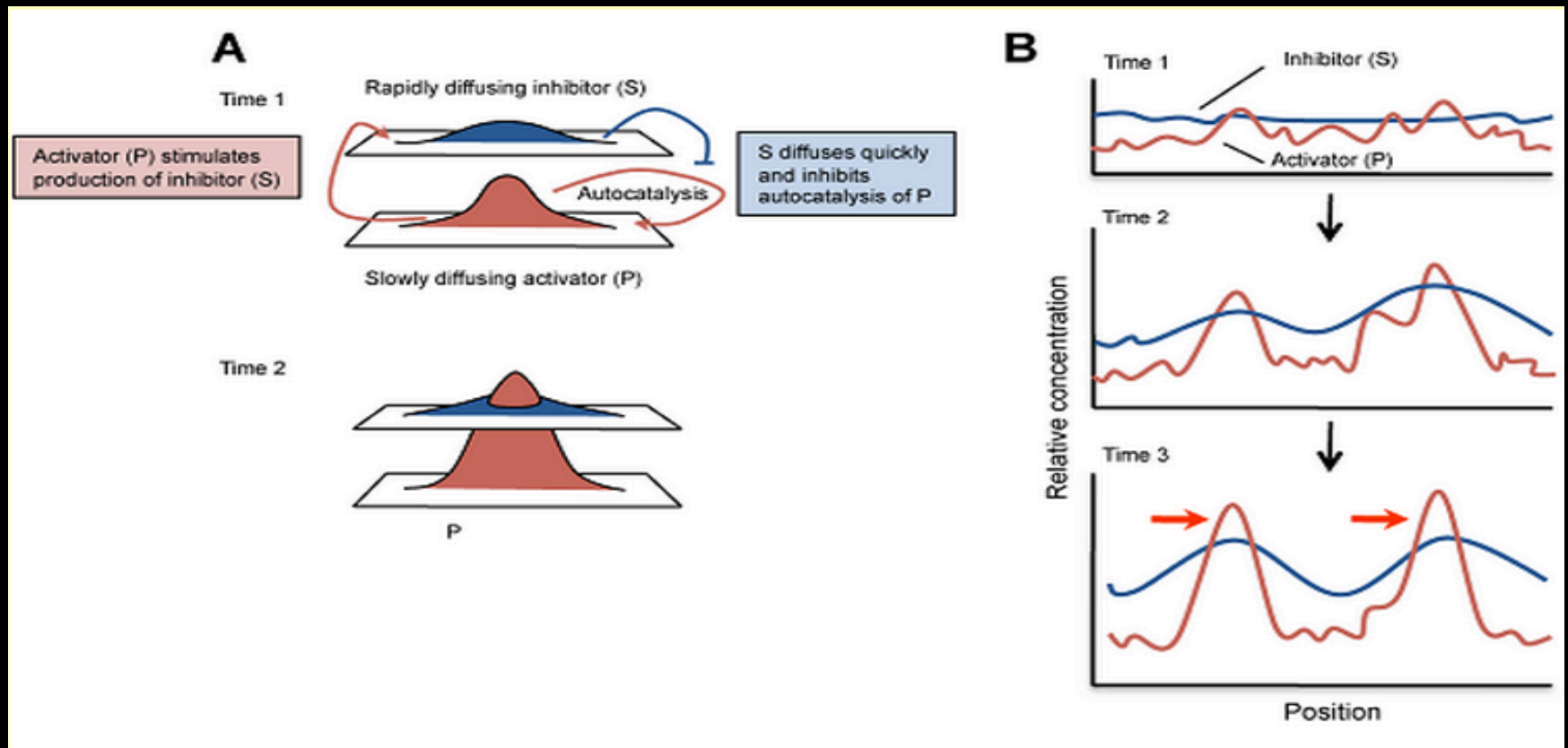
# J.D. Murray's Fire and Grasshopper metaphor



Source Philip Maini's talk on youtube:  
<http://www.youtube.com/watch?v=pN8tVldm6QY>

# Basic principles behind patterning

- Short-range activation gives long-range inhibition

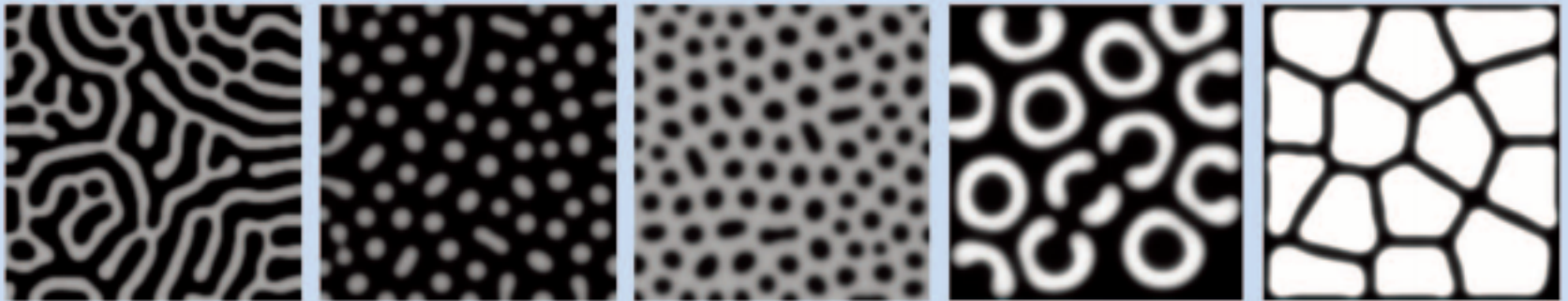


# Predictions from Turing's Model

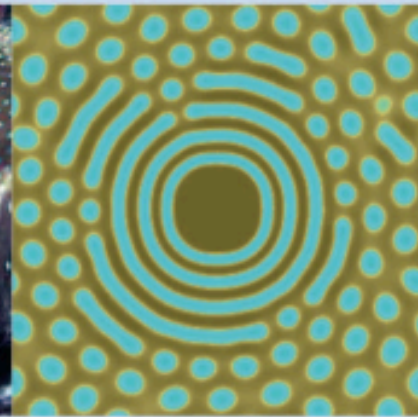
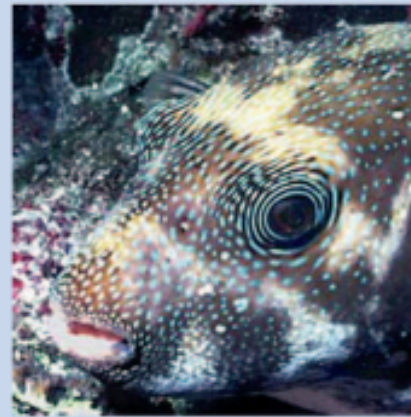
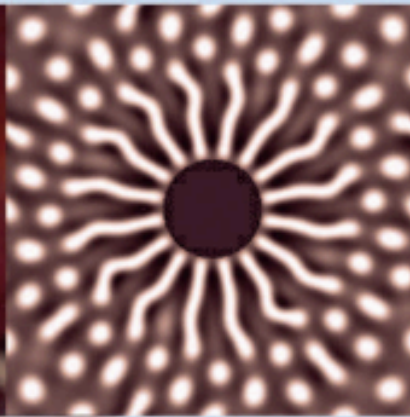
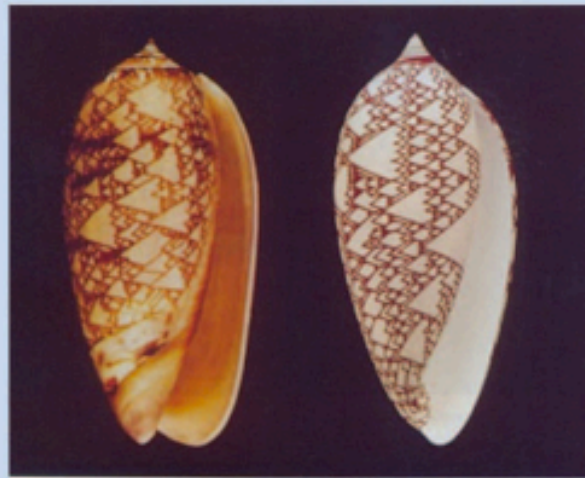
- For Turing patterns to occur, the spatial domain should have a minimum size
- Complexity of the Turing patterns is dependent on the domain size.
- Geometry of the spatial domain plays a big role



Case VI (Turing pattern)



Source: *Reaction-Diffusion Model as a Framework for Understanding Biological Pattern Formation* Shigeru Kondo and Takashi Miura  
Science 329, 1616 (2010);



From: *Reaction-Diffusion Model as a Framework for Understanding Biological Pattern Formation* Shigeru Kondo and Takashi Miura  
Science 329, 1616 (2010);



# How the Leopard got its spots?



Source: Google images



$$\frac{\partial u}{\partial t} = \gamma f(u, v) + D_u \nabla^2 u,$$

$$\frac{\partial v}{\partial t} = \gamma g(u, v) + D_v \nabla^2 v,$$

$$f(u, v) = a - u - h(u, v),$$

$$g(u, v) = \alpha(b - v) - h(u, v),$$

$$h(u, v) = \frac{\rho uv}{1 + u + Ku^2},$$



Source: J. D. Murray's article, <http://www.ams.org/notices/201206/rtx120600785p.pdf>

Spotted animals can have striped tails but not the other way round

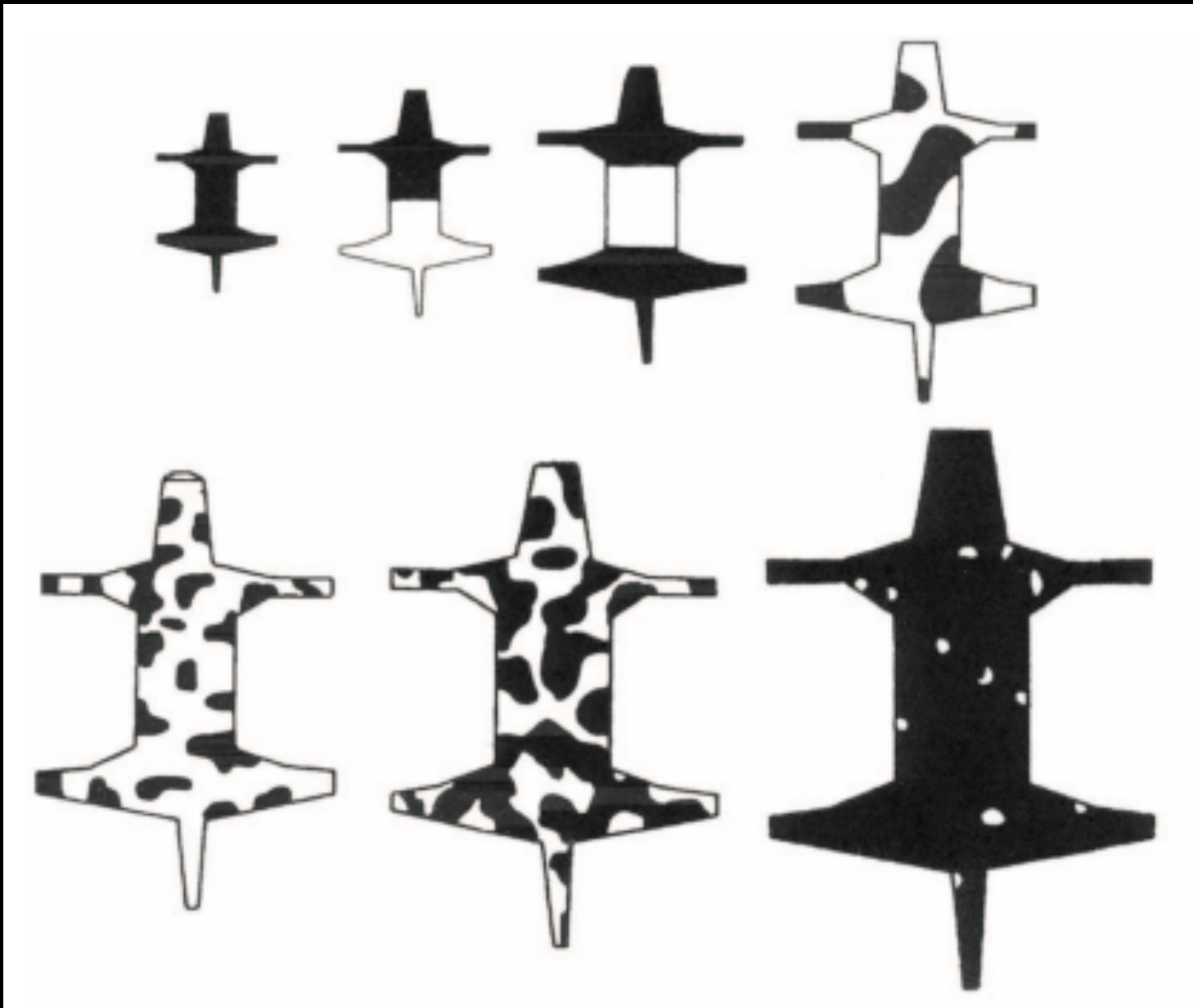




**“All Models are wrong... but  
some are useful” – George Box**

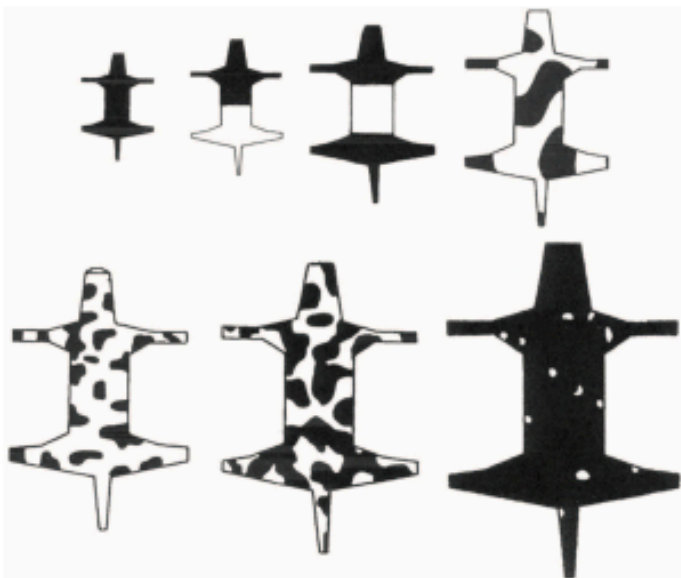


Source Philip Maini's talk on youtube:  
<http://www.youtube.com/watch?v=pN8tVldm6QY>



Source: J. D. Murray's article, <http://www.ams.org/notices/201206/rtx120600785p.pdf>





(a)



(b)

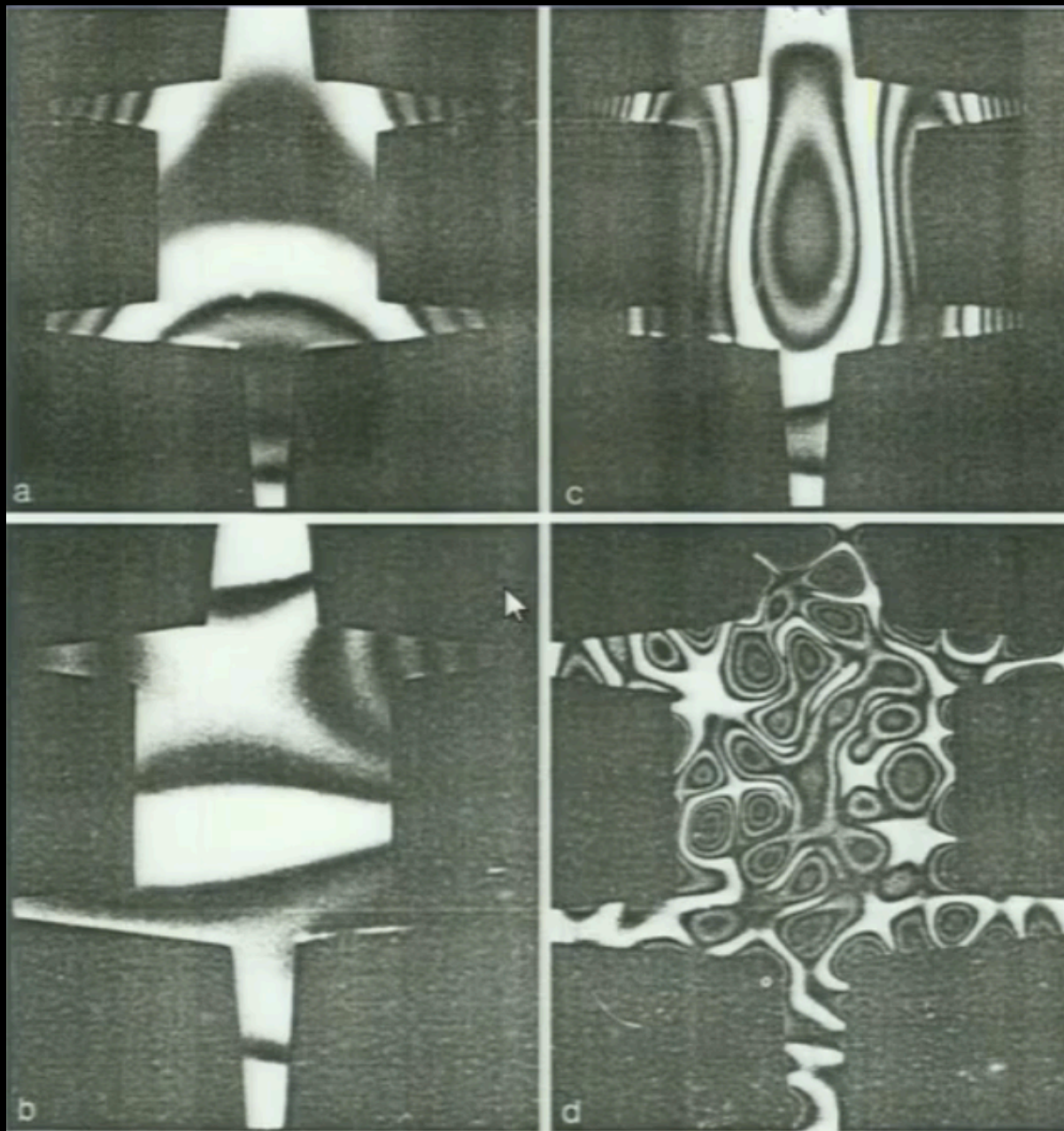


(c)



(d)





Source Philip Maini's talk on youtube:  
<http://www.youtube.com/watch?v=pN8tVldm6QY>

# Conclusions

- Turing's paper on morphogenesis revolutionized biology
- Still remains a seminal work in several areas
- Inspired an enormous amount of mathematics.