

Turing Machines

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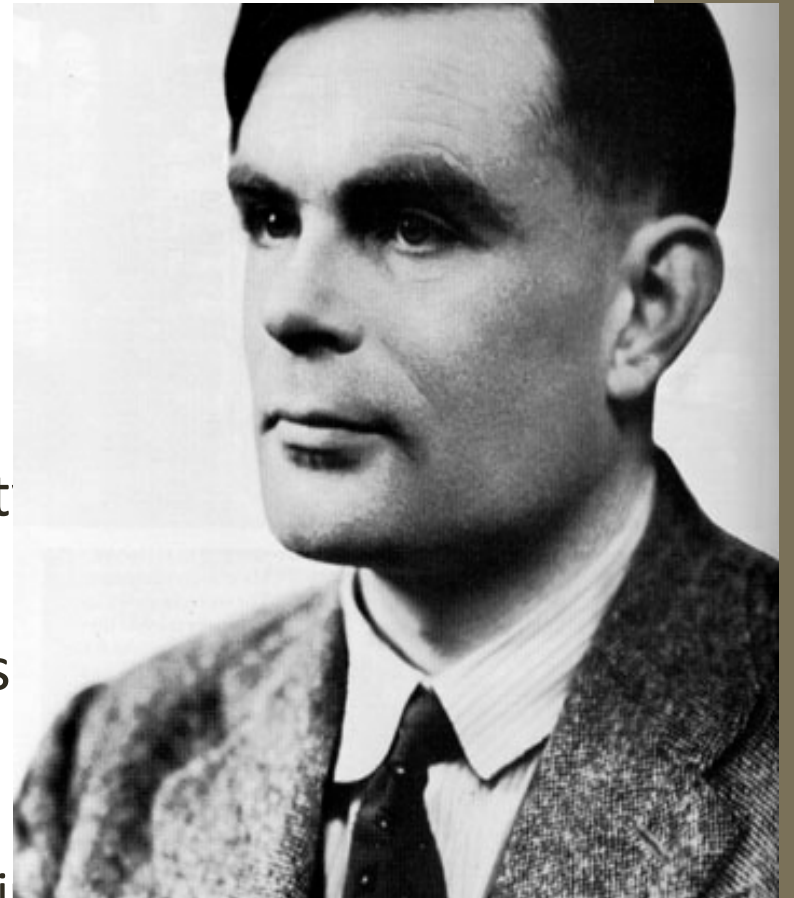
Early Computers



Early NACA human computers at work (1949)

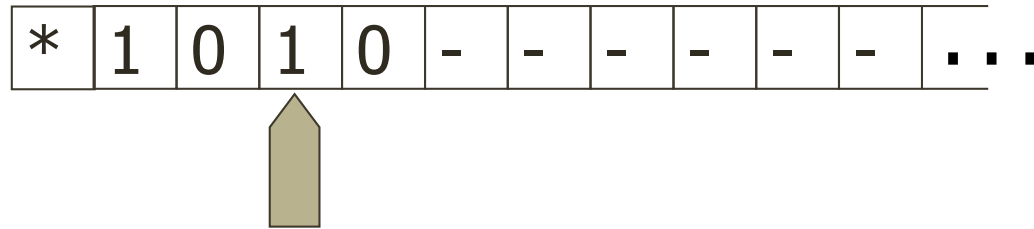
Effective Process

- Can be done with pencil and paper.
- Is a finite set of instructions.
- Demands neither insight or ingenuity.
- Will definitely work without error.
- Produces in a finite number of steps either:
 - A final result, or
 - If the result is a sequence, each symbol in the sequence.



Turing Machine

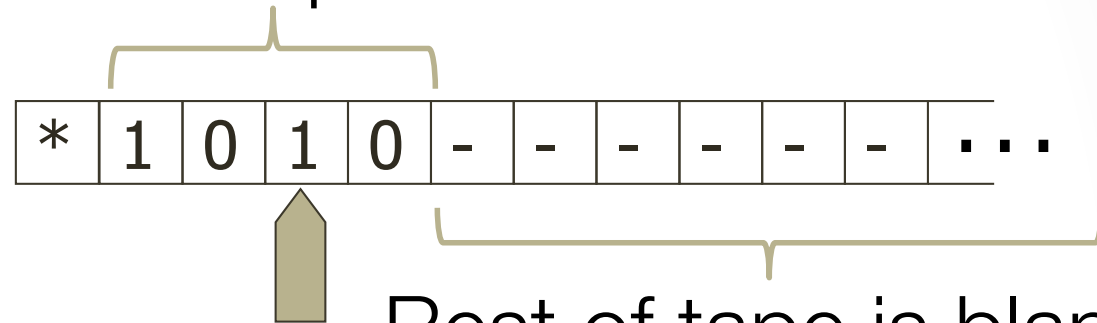
- A tape and tape head.



- A Program
 - A sequence of instructions

The Tape

Finite Input

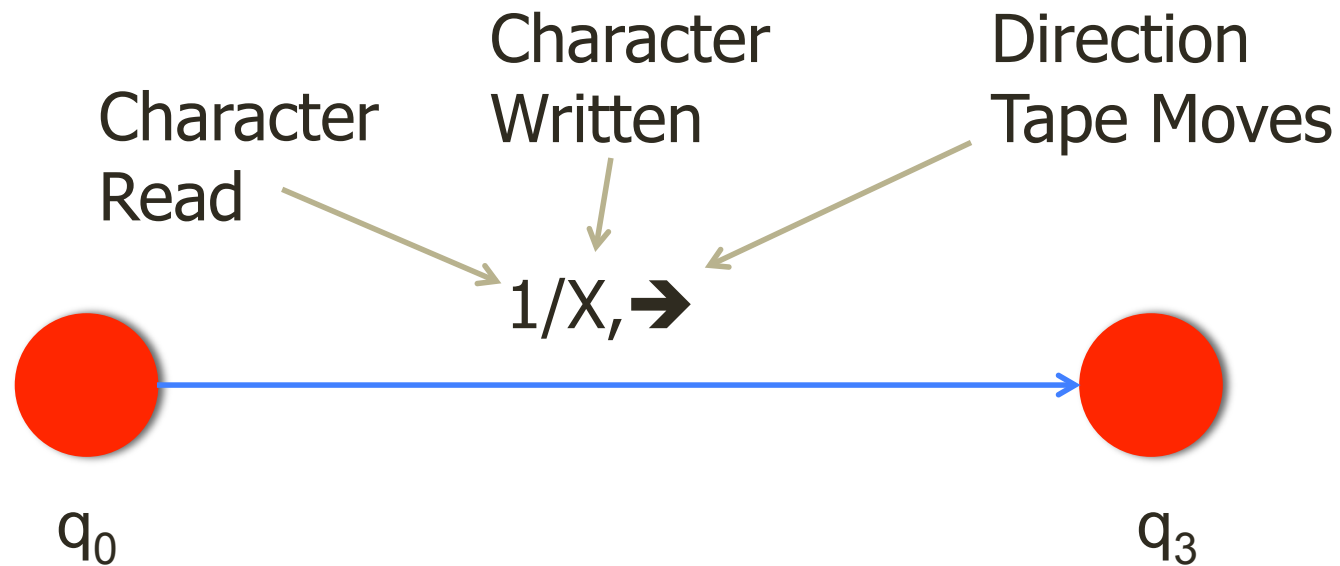


Rest of tape is blanks

- Tape is infinite in one direction.
- Tape head can move left and right.
- Tape head can read and write characters.

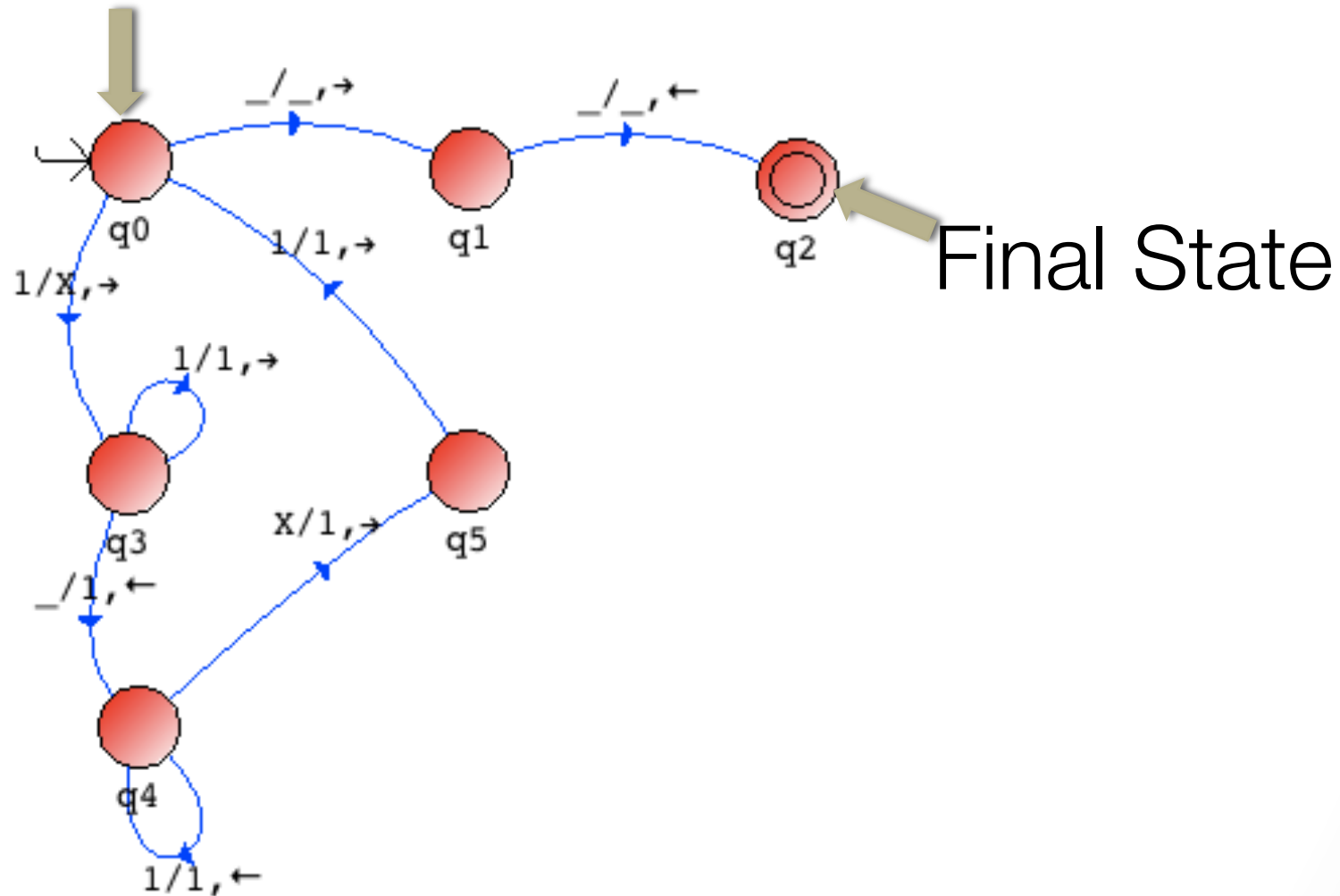
Instructions

If (in state q_0 and reading 1) then
write X in the current square
move right one square on the tape
goto state q_3



Program

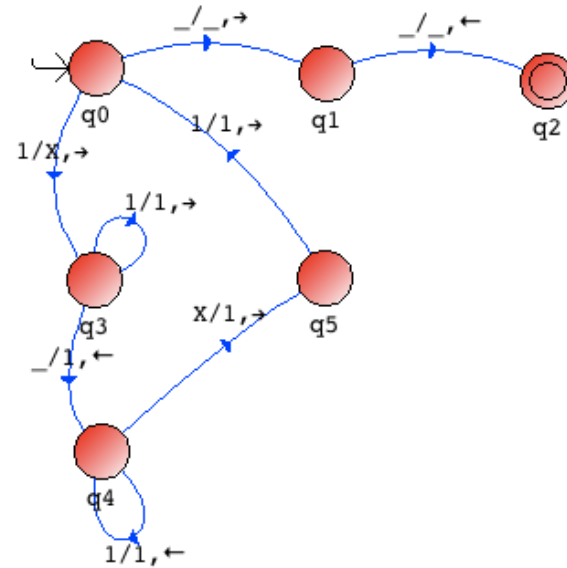
Start State



Representation of Integers

Integer	Representation
0	Blank Tape
1	1
2	11
3	111
4	1111
5	11111
6	111111
7	1111111
8	11111111

A Function



Input	Output
Blank Tape	Blank Tape
1	11
11	1111
1...1	1...11...1

N
 $2N$

$$F(N) = 2N$$

Turing-Church Thesis

***Every effective procedure can be performed
by a Turing Machine.***

Assumptions

- `_` will represent a blank
- We always start in state `q0`
- We always end in state `q2`
- We will assume that the tape head starts on the first square.
- Alphabet is: `0, 1, X, _`

Representing Instructions

Current State	Character Read	Character Written	Direction Tape Moves	Next State
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Represent as an integer



Represent as an integer

Action/Character	Representation
Move Left	0
Move Right	1
0	000
1	001
—	010
X	011

Representation

Current State	Character Read	Character Written	Direction Tape Moves	New State	
q ₀	–	–	→	q ₁	01001011
q ₀	1	X	→	q ₃	0010111111
q ₁	–	–	←	q ₂	1010010011
q ₃	1	1	→	q ₃	1110010011111
q ₃	–	1	←	q ₄	11101000101111
q ₄	1	1	←	q ₄	111100100101111
q ₄	X	1	→	q ₅	1111011001111111
q ₅	1	1	→	q ₀	111110010011

**010010110001011111101010010011011100100111110111010001011110111100
100101111011110110011111110111110010011**

Universal Turing Machine (UTM)

- A Turing Machine which can simulate the behaviour of any Turing Machine on any data.
- A UTM is a representation of the concept of a programmable computer.